JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 18, No. 1, January–February 1995

Hybrid Near-Optimal Atmospheric Guidance for Aeroassisted Orbit Transfer

Michael B. McFarland and Anthony J. Calise*
Georgia Institute of Technology, Atlanta, Georgia 30332

In this paper, a hybrid methodology is applied to the problem of determining a near-optimal control which approximately minimizes energy loss for a class of aeroassisted orbit transfer maneuvers. The result is an approximate solution incorporating aspects of both an analytical zero-order solution from regular perturbation and a numerical solution from collocation. This approach is used to compute both open-loop and closed-loop near-optimal controls for a particular vehicle. Results from several approximations are quantitatively compared with an exact optimal solution.

Introduction

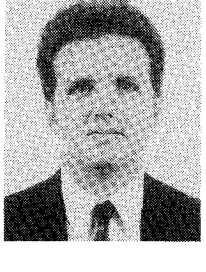
An orbit plane change is one of the most fuel-intensive maneuvers that can be required of an orbiting space vehicle. It has been shown that an aeroassisted orbit transfer consumes significantly less fuel than a similar maneuver executed with thrusters only. This advantage is only realizable, however, when optimal control laws are developed that take full advantage of the available aerodynamic forces. Because the dynamics governing this type of maneuver are highly nonlinear, an analytical solution to the optimal control problem is impossible. Furthermore, an exact numerical solution is too computationally demanding for real-time implementation onboard the orbiting vehicle. Approximation techniques that allow near-optimal guidance laws to be determined either analytically or with a minimum of computational effort are, therefore, highly desirable for this application. In this paper, a method is considered that combines elements of both approaches. An analytical approximation that holds for the atmospheric phase of flight is obtained from the zero-order solution to a regular perturbation problem. This solution is then augmented numerically by a technique borrowed from the method of collocation, in which the trajectory is divided into finite elements.

In recent efforts, numerous approximate methods have been applied to the task of determining a near-optimal control for aeroassisted orbit transfer. These approximations have addressed the atmospheric portion of the maneuver, assuming that the effects of the orbital dynamics are small.[1–3] Keplerian effects are treated by introducing an approximation of Loh's term. Some approximations of this type have included assuming that Loh's term is constant or piecewise constant.[2] A second alternative is to account for Keplerian effects through a regular perturbation expansion of the control solution.[1] A related approach to the problem addresses both the orbital and atmospheric stages of the trajectory by invoking singular perturbation theory and matched asymptotic expansions.[4] It has been shown in Ref. 4 that the problem in question is, in fact, a singularly perturbed problem and that variations in Loh's term can be accounted for by properly matching a zero-order outer solution with the zero-order solution used in Ref. 1. That is, the zero-order solution of Ref. 1 serves as the inner solution in a matched asymptotic expansion.

The paper begins with a review of the dynamics of aeroassisted orbit transfer and the associated optimal control problem. A summary of the problem formulation as presented in Ref. 1 is included in the interests of clarity and completeness. The problem definition is followed by a description of its analytical zero-order solution. An overview of the method of collocation is then presented as an entirely numerical methodology for this problem. Next, the hybrid methodology incorporating elements of both solutions is described. This method was originally developed in Ref. 5 and applied to the problem of launch vehicle guidance. Finally, numerical results are presented for the maneuverable research re-entry vehicle (MRRV) which was featured in Ref. 3. These results are discussed in detail with recommendations for further analysis.

Aeroassisted Orbit Transfer Problem

The aeroassisted orbit transfer maneuver may be defined as an optimal control problem. This problem is characterized by highly nonlinear coupled differential constraints and is not amenable to an analytical solution. The equations may, however, be transformed and then used under certain simplifying assumptions to determine a near-optimal control which is valid for the atmospheric portion of the maneuver. In this section, the equations of motion which govern this type of maneuver are presented, and a scheme for modifying the equations is reviewed. The associated optimal control problem is then defined, along with the general form of its solution.



Michael McFarland is a research engineer with the U.S. Air Force Armament Directorate. He is currently pursuing his Ph.D. in aerospace engineering at the Georgia Institute of Technology under the Air Force's "Palace Knight" career program. He received a B.S.E. degree in aerospace engineering from the University of Florida in 1991 and an M.S. degree in aerospace engineering from the Georgia Institute of Technology in 1992. His research interests include adaptive and/or nonlinear control, neural networks, and control of guided weapon systems. He is a Student Member of AIAA.

Received Sept. 9, 1993; revision received June 3, 1994; accepted July 7, 1994. Copyright © 1994 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved.

*Professor, School of Aerospace Engineering. Fellow AIAA.

Equations of Motion

The dynamical equations for a particle of mass m in motion about a spherical nonrotating planet are as follows:

$$\frac{dr}{dt} = V \sin\gamma \tag{1}$$

$$\frac{dV}{dt} = -\frac{D}{m} - g \sin\gamma \tag{2}$$

$$\frac{d\gamma}{dt} = \frac{L\cos\mu}{mV} - \left(\frac{g}{V} - \frac{V}{r}\right)\cos\gamma \tag{3}$$

$$\frac{d\psi}{dt} = \frac{L\sin\mu}{mV\cos\gamma} - \frac{V}{r}\cos\gamma\cos\psi\tan\phi \tag{4}$$

$$\frac{d\phi}{dt} = \frac{V\cos\gamma\sin\psi}{r} \tag{5}$$

$$\frac{d\theta}{dt} = \frac{V\cos\gamma\cos\psi}{r\cos\phi} \tag{6}$$

where r is the distance from the center of mass of the planet to the point mass in motion, V is its speed, γ is the flight-path angle, ψ is the heading angle, ϕ is the crossrange angle, θ is the downrange angle, μ is the bank angle, t is time, and g is the local gravitational acceleration.

To facilitate analysis, it is necessary to modify these general equations through the introduction of several transformations and assumptions. The modified problem formulation employed in this analysis was presented in Ref. 1 and will be reviewed here. This formulation has the primary advantage of isolating the Keplerian portion of the vehicle's motion, which may be considered to have a perturbing effect in the dynamic equations throughout the atmospheric phase of the aeroassisted orbit transfer maneuver. First, it is necessary to model explicitly certain quantities which appear in the equations of motion. The gravitational field is modeled by an inverse square law of the form

$$g = \bar{\mu}/r^2 \tag{7}$$

where $\bar{\mu}$ is the planetary gravitation constant. The atmospheric density ρ is modeled locally by the exponential relation

$$\rho = \rho_s e^{-[(r-r_s)/\beta]} \tag{8}$$

where ρ_s is a reference density, r_s is the reference radius, and β is the exponential atmospheric scale height. In modeling the aerodynamic forces, the following simple relationships are employed:

$$L = \tfrac{1}{2}\rho V^2 S C_L$$
$$D = \tfrac{1}{2}\rho V^2 S C_D \tag{9}$$
$$C_D = C_{D_0} + K C_L^2$$

where L is the lift force, D is the drag force, C_L is the lift coefficient, C_D is the drag coefficient, C_{D_0} is the drag coefficient at zero lift, S is the aerodynamic surface area, and K is a constant coefficient in the parabolic drag polar.

Nondimensional variables are defined by the following transformations:

$$w = \frac{C_L^* \rho S \beta}{2m} \qquad \lambda = \frac{C_L}{C_L^*}$$
$$\nu = \ln\left(\frac{V^2}{gr}\right) \qquad \delta = \lambda\cos\mu \tag{10}$$
$$E^* = \frac{C_L^*}{C_D^*} \qquad \sigma = \lambda\sin\mu$$

where w and ν are the nondimensional variables related to altitude and velocity, respectively, and λ is the normalized lift coefficient. E^* denotes the maximum lift-to-drag ratio, and C_L^* and C_D^* are defined as follows:

$$C_D^* = 2C_{D_0} = 2KC_L^{*2} \tag{11}$$

Finally, δ and σ are the control variables corresponding to the local vertical and horizontal components of normalized lift.

A feature of the formulation presented in Ref. 1 which is not fully exploited in this analysis is the use of a new monotonically increasing independent variable z defined by the transformation

$$\frac{d\tau}{dz} = \frac{\beta}{wV} = \frac{mV}{L^*} \tag{12}$$

One motivation for this choice of independent variable will subsequently become evident. Moreover, to isolate the orbital portions of the equations of motion a small parameter which multiplies each of these Keplerian terms is identified as

$$\varepsilon = \beta/r_s \tag{13}$$

Neglecting this small parameter is equivalent to assuming that atmospheric effects dominate the orbital dynamics. This is also equivalent to assuming that Loh's term, defined as

$$\bar{M} = \frac{2m}{C_L^* S}\left[\frac{1-\bar{\mu}/(V^2 r)}{\rho r}\right] \tag{14}$$

in Ref. 2, is identically zero. This is a reasonable approximation in the atmospheric portion of the aeroassisted orbit transfer as long as velocity remains close to the circular velocity and the flight-path angle is small. This results in an integrable system of differential equations. Finally, the flight-path angle is assumed to be small so that $\sin\gamma \approx \gamma$ and $\cos\gamma \approx 1$.

In Ref. 2 it is shown that, for short duration maneuvers, the change in crossrange angle ϕ is small, and the change in inclination is closely approximated by the change in heading. Furthermore, in Ref. 6 it is shown that although the actual inclination change depends on the initial inclination, the starting point of the maneuver can be timed so as to obtain the maximum inclination change that is achieved when the initial inclination is zero. A consequence of this result is that the initial plane can be defined as the equatorial plane. Under these assumptions, the initial values of θ and ϕ can be set equal to zero without loss of generality. Thus, θ and ϕ become ignorable coordinates. The resulting equations of motion are

$$\frac{dw}{dz} = -\gamma \tag{15}$$

$$\frac{d\nu}{dz} = -\frac{(1+\sigma^2+\delta^2)}{E^*} - \varepsilon\left[\frac{r_s(2e^{-\nu}-1)\gamma}{rw}\right] \tag{16}$$

$$\frac{d\gamma}{dz} = \delta + \varepsilon\left[\frac{r_s(1-e^{-\nu})}{rw}\right] \tag{17}$$

$$\frac{d\psi}{dz} = \sigma \tag{18}$$

Note that the independent variable z and the small parameter ε have been chosen so that Eqs. (15–18) have the general form

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u}) + \varepsilon\boldsymbol{g}(\boldsymbol{x}) \tag{19}$$

where $\boldsymbol{x}$ and $\boldsymbol{u}$ are state and control vectors, respectively. The $\varepsilon = 0$ approximation yields a dynamic model for the zero-order problem in a regular perturbation analysis and will hereafter be referred to as such. This formulation has the feature that the choice of independent variable is such that the controls do not explicitly appear in the perturbation dynamics. Its advantage lies in the calculation of first- and higher order correction terms which may be used, as in Ref. 1, to improve the zero-order approximation. In this paper, however, higher order corrections are omitted, and a technique borrowed from the numerical method of collocation is used to correct the zero-order approximation for variations in Loh's term. Use of this form will simplify extension to a first-order analysis if one is desired.

Optimal Control Formulation

The optimal control problem associated with the aeroassisted orbit transfer maneuver is to minimize the energy loss or, equivalently, the negative of the final velocity, subject to Eqs. (15–18) with prescribed boundary conditions. These boundary conditions include initial conditions which specify the values w_i, ν_i, γ_i, and ψ_i as well as terminal conditions specifying w_f, γ_f, and ψ_f. The Hamiltonian for this problem is given by

$$H = \lambda_\psi \sigma - \lambda_w \gamma + \lambda_\gamma \left[\delta + \varepsilon\left(\frac{r_s(1-e^{-\nu})}{rw}\right)\right] - \lambda_\nu\left[\frac{(1+\sigma^2+\delta^2)}{E^*} + \varepsilon\left(\frac{r_s(2e^{-\nu}-1)\gamma}{rw}\right)\right] \tag{20}$$

where λ_w, λ_ν, λ_γ, and λ_ψ are Lagrange multipliers, or costates. The augmented end-point function for this problem is

$$G = -\nu(z_f) + \pi_w[w(z_f) - w_f] + \pi_\gamma[\gamma(z_f) - \gamma_f] + \pi_\psi[\psi(z_f) - \psi_f] \tag{21}$$

where π_w, π_γ, and π_ψ are Lagrange multipliers. The final value of the independent variable is denoted by z_f and satisfies the transversality condition $H(z_f) = 0$.

The optimality conditions

$$H_\delta = \lambda_\gamma - \frac{2\delta\lambda_\nu}{E^*} = 0 \qquad H_\sigma = \lambda_\psi - \frac{2\sigma\lambda_\nu}{E^*} = 0 \tag{22}$$

yield expressions for the vertical and horizontal lift force controls

$$\delta = \frac{E^*\lambda_\gamma}{2\lambda_\nu} \qquad \sigma = \frac{E^*\lambda_\psi}{2\lambda_\nu} \tag{23}$$

independent of ε.

Analytical Zero-Order Approximation

In Ref. 1, an analytical solution is presented for the zero-order problem associated with the regular perturbation formulation in Eqs. (15–18). The zero-order dynamics are

$$\frac{dw_0}{dz} = -\gamma_0 \tag{24}$$

$$\frac{d\nu_0}{dz} = -\frac{\left(1+\sigma_0^2+\delta_0^2\right)}{E^*} \tag{25}$$

$$\frac{d\gamma_0}{dz} = \delta_0 \tag{26}$$

$$\frac{d\psi_0}{dz} = \sigma_0 \tag{27}$$

The Hamiltonian for the zero-order problem is

$$H_0 = \lambda_{\psi 0}\sigma_0 - \lambda_{w0}\gamma_0 - \lambda_{\nu 0}\frac{\left(1+\sigma_0^2+\delta_0^2\right)}{E^*} + \lambda_{\gamma 0}\delta_0 \tag{28}$$

and the end-point function is given by Eq. (21). The necessary conditions for optimality lead to the following expressions for the optimal controls:

$$\delta_0 = E^*\lambda_{\gamma 0}/(2\lambda_{\nu 0}) \tag{29}$$

$$\sigma_0 = \frac{E^*\lambda_{\psi 0}}{2\lambda_{\nu 0}} \tag{30}$$

where $\lambda_{\psi 0}$ and $\lambda_{\nu 0}$ are constant and, therefore, σ_0 is constant. As shown in Ref. 1, the optimal constant horizontal force σ_0 is given by a root of the fourth-order polynomial

$$C_4\sigma_0^4 + C_3\sigma_0^3 + C_2\sigma_0^2 - C_0 \tag{31}$$

in which the constant coefficients are

$$\begin{gathered} C_0 = (\Delta\psi)^4 \\ C_2 = \left[(\Delta\psi)^2 + (\Delta\gamma)^2 + 3(\gamma_f+\gamma_i)^2\right](\Delta\psi)^2 \\ C_3 = 24(\gamma_f+\gamma_i)\Delta w\Delta\psi; \qquad C_4 = 36(\Delta w)^2 \end{gathered} \tag{32}$$

where $\Delta w = (w_f - w_i)$, $\Delta\gamma = (\gamma_f - \gamma_i)$, and $\Delta\psi = (\psi_f - \psi_i)$.

The optimal state and costate histories are described by

$$\begin{gathered} w_0(z) = \frac{E^*\lambda_{w0}}{12}z^3 - \left(\frac{\Delta\gamma}{2z_f} + \frac{E^*\lambda_{w0}z_f}{8}\right)z^2 - \gamma_i z + w_i \\ \nu_0(z) = -\lambda_{w0}(w_0 - w_i) - (2/E^*)z + \nu_i \\ \gamma_0(z) = -\frac{E^*\lambda_{w0}}{4}z^2 + \left(\frac{\Delta\gamma}{z_f} + \frac{E^*\lambda_{w0}z_f}{4}\right)z + \gamma_i \\ \psi_0(z) = \sigma_0 z + \psi_i \\ \lambda_{w0}(z) = -\frac{24}{E^*z_f^2}\left[\frac{\Delta w}{z_f} + \frac{(\gamma_i+\gamma_f)}{2}\right] \\ \lambda_{\nu 0}(z) = -1 \\ \lambda_{\gamma 0}(z) = \lambda_{w0}\left(z - \frac{z_f}{2}\right) - \frac{2\Delta\gamma}{E^*z_f} \\ \lambda_{\psi 0}(z) = -(2\sigma_0/E^*) \end{gathered} \tag{33}$$

These histories will be illustrated for a particular vehicle, the MRRV featured in Ref. 3.

Numerical Collocation Solution

In contrast to the approach of Ref. 1 just outlined, it is possible to generate an approximate solution to the optimal control problem by entirely numerical methods. What follows is a brief discussion of the numerical method of collocation.

The method of collocation is a numerical technique in which the solution to a system of differential equations is constructed by dividing the problem into finite elements. Within each element, the solution is described by simple analytical interpolating functions, often polynomials with undetermined coefficients. Constraint equations introduced to determine these coefficients include continuity constraints applied at each node, as well as derivative constraints which ensure that the interpolated solution satisfies the differential equations at specified points within each element.

As an example, consider a general optimal control problem governed by differential constraints $dx/dt = f(x, u, t)$ with Hamiltonian $H = \boldsymbol{\lambda}^T \boldsymbol{f}$. Although higher order polynomials such as cubic splines are often preferred, it is assumed for discussion purposes that linear interpolating functions are employed in the collocation formulation and that the differential constraints are applied only at the midpoint of each element. In this case, the piecewise linear approximate solution has the form

$$\left.\begin{aligned} \boldsymbol{x}(t) &= \boldsymbol{x}_{j-1} + \boldsymbol{p}_j(t - t_{j-1}) \\ \boldsymbol{\lambda}(t) &= \boldsymbol{\lambda}_{j-1} + \boldsymbol{q}_j(t - t_{j-1}) \end{aligned}\right\} \begin{aligned} & j = 1, \ldots, N \\ & t \in [t_{j-1}, t_j] \end{aligned} \tag{34}$$

where $t_0 = t_i$, $t_N = t_f$, and N is the number of elements. After using the optimality condition to eliminate the control from the system dynamics, the derivative constraints take the form

$$p_j = \frac{x_j - x_{j-1}}{t_j - t_{j-1}} = \frac{\partial H}{\partial \lambda}\Bigg|_{\substack{t=(t_j+t_{j-1})/2\\ x=(x_j+x_{j-1})/2\\ \lambda=(\lambda_j+\lambda_{j-1})/2}}$$

$$q_j = \frac{\lambda_j - \lambda_{j-1}}{t_j - t_{j-1}} = -\frac{\partial H}{\partial x}\Bigg|_{\substack{t=(t_j+t_{j-1})/2\\ x=(x_j+x_{j-1})/2\\ \lambda=(\lambda_j+\lambda_{j-1})/2}} \tag{35}$$

The approximate solution may be evaluated either by computing the unknown coefficients (p_j, q_j) or by determining the nodal values of the states and costates directly.

The method of collocation is readily applied to the aeroassisted orbit transfer problem. In this paper, a simple Newton search is used to calculate the nodal values of the states and costates.[7] Various strategies were considered for spacing the elements since the nondimensional formulation employed here and in Ref. 1 exhibits more rapid variations in the latter portion of the trajectory. Note that the problem is characterized by four states and four costates, resulting in $8N$ derivative constraints. The initial boundary conditions may be directly enforced, leaving three prescribed final boundary conditions, one natural boundary condition, and the transversality condition. This corresponds to a system of $8N + 5$ algebraic equations, which must be solved for the $8N$ unknown coefficients (p_j, q_j), the four costate initial values, and the final value of the independent variable z. Numerical results obtained by collocation for the MRRV from Ref. 3 will be presented.

Hybrid Solution: Zero-Order Approximation with Collocation

In Ref. 5, a procedure is outlined in which the collocation solution is in fact the zero-order solution to a regular perturbation problem. If piecewise linear dynamics closely approximate the true dynamics, then an error term may be introduced, and the actual system of differential equations may be written as

$$\left.\begin{aligned} \frac{dx}{dt} &= p_j + \varepsilon_1\left(\frac{\partial H}{\partial \lambda} - p_j\right) \\ \frac{d\lambda}{dt} &= q_j + \varepsilon_1\left(-\frac{\partial H}{\partial x} - q_j\right) \end{aligned}\right\} \begin{aligned} & j = 1, \dots, N \\ & t \in [t_{j-1}, t_j]; \quad H_u = 0 \end{aligned} \tag{36}$$

where the Hamiltonian H corresponds to the original system without a perturbation parameter. Here, the parameter ε_1 is nominally equal to 1.0, but if the error dynamics are small, they may be considered to have a perturbing effect, which is exactly zero where the derivative constraints are enforced. A zero-order solution can be determined by letting $\varepsilon_1 = 0$, and higher order corrections may be computed from regular perturbation theory.

The formulation just presented not only facilitates the improvement of the collocation solution by means of higher order correction, but also allows accuracy to be increased by moving analytically tractable portions of the dynamics out of the perturbation terms and bringing these dynamics into evidence in the zero-order solution. The latter approach is intended to reduce the number of elements required to obtain a satisfactory approximation of the optimal control and, possibly, to eliminate the need for the computation of higher order corrections. When this technique is applied to a system of the form of Eq. (19) the result is the equivalent system

$$\frac{dx}{dt} = f(x, u) + p_j + \varepsilon_1[\varepsilon g(x) - p_j] \tag{37}$$

with the costate derivatives taking on a similar form. This formulation is essentially collocation with complicated interpolation functions which are based on the zero-order dynamics of the system.

For the aeroassisted orbit transfer problem, Eqs. (15–18) along with the optimality conditions suggest that the system dynamics may be rewritten as

$$\begin{aligned} \frac{dw}{dz} &= -\gamma \\ \frac{d\nu}{dz} &= -\frac{(1+\sigma^2+\delta^2)}{E^*} - \varepsilon_1\left[\frac{\beta(2e^{-\nu}-1)\gamma}{rw}\right] \\ \frac{d\gamma}{dz} &= \delta + p_{\gamma_j} + \varepsilon_1\left[\frac{\beta(1-e^{-\nu})}{rw} - p_{\gamma_j}\right] \\ \frac{d\psi}{dz} &= \sigma \\ \frac{d\lambda_w}{dz} &= q_{w_j} + \varepsilon_1\left(-\frac{\partial H}{\partial w} - q_{w_j}\right) \\ \frac{d\lambda_\nu}{dz} &= 0 + \varepsilon_1\left(-\frac{\partial H}{\partial \nu}\right) \\ \frac{d\lambda_\gamma}{dz} &= \lambda_w + q_{\gamma_j} + \varepsilon_1\left(-\frac{\partial H}{\partial \gamma} - \lambda_w - q_{\gamma_j}\right) \\ \frac{d\lambda_\psi}{dz} &= 0 \end{aligned} \tag{38}$$

for all $z \in [z_{j-1}, z_j]$, where the controls σ and δ satisfy Eq. (22), and H is the Hamiltonian from Eq. (20). The definitions of w and ρ imply that

$$r = r_s - \beta \ln\left(\frac{2mw}{C_L^* \rho_s \beta S}\right) \tag{39}$$

Note that since the state dynamics with the exception of flight-path angle are closely approximated in the original zero-order system, no additional interpolating constants (p_j) are introduced in the w, ν, and ψ equations. It is further assumed that the λ_ν dynamics are approximated sufficiently well in the original zero-order system. These assumptions yield solutions to Eqs. (38) which are polynomials of moderate order in z. These interpolating functions are included in the Appendix.

To calculate a zero-order solution, the approximation $\varepsilon_1 = 0$ is introduced, with $\varepsilon = \beta/r_s$ maintained at its nominal value in Eq. (20). This means that the actual Hamiltonian is used when enforcing the derivative constraints at the midpoints of the elements. Note that in this formulation the collocation constraints, prescribed final conditions, and transversality condition result in a system of only $3N + 4$ algebraic equations for the $3N$ undetermined constants (p_{γ_j}, q_{w_j}, and q_{γ_j}), three initial costate values, and the final value of the independent variable z. Since λ_ν is approximated as a constant, the natural boundary condition may be directly enforced. Numerical results obtained for the MRRV using this method are included in the following section.

Numerical Example

The techniques described previously may be used to determine near-optimal open-loop guidance laws for a particular vehicle executing typical aeroassisted orbit transfer maneuvers. Optimal solutions which serve as a standard for evaluating the approximations were generated using multiple shooting software.[8]

The vehicle considered here is the maneuverable research reentry vehicle and is described in Ref. 3. The physical constants are $K = 1.4$, $C_{D_0} = 0.032$, $S = 11.69$ m^2, and $m = 4898.7$ kg. The modeling parameters include the exponential atmospheric scale height $\beta = 8251.585$ m, reference radius $r_s = 6.433375E6$ m, reference density $\rho_s = 5.6075E$-4 kg/m^3, and the gravitational constant $\bar{\mu} = 3.98603E14$ m^3/s^2. Initial conditions for all simulations are $H_i = 60$ km, $V_i = 7850.88$ m/s, $\gamma_i = -1.346$ deg, and $\psi_i = 0$ deg. The specified terminal conditions are $H_f = 60$ km, $\gamma_f = 1.00$ deg, and $\psi_f = 20$ deg. The results presented here compare the zero-order analytic solution, the piecewise linear collocation solution, and the hybrid solution described previously with the true optimal solution.

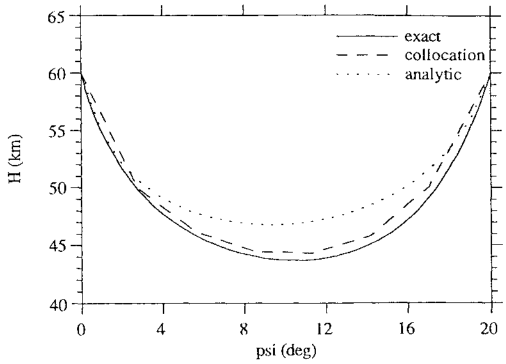


Fig. 1 Open-loop *H* profiles.

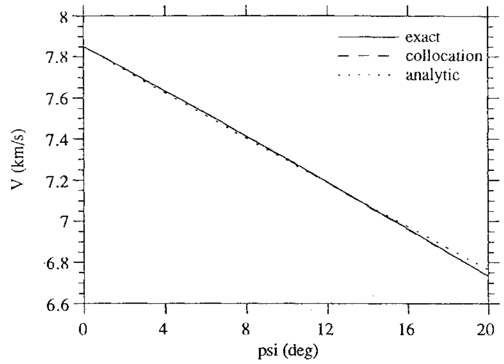


Fig. 2 Open-loop *V* profiles.

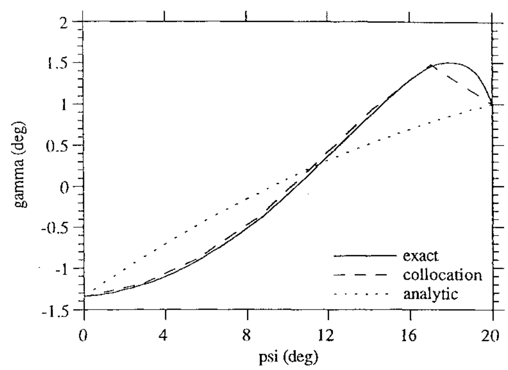


Fig. 3 Open-loop γ profiles.

The data presented in Figs. 1–3 includes open-loop results obtained from the purely analytical zero-order solution as well as the numerical method of collocation with $N = 7$ piecewise linear interpolating functions. The piecewise linear nature of the collocation solutions is evident in these plots.

Clearly, the zero-order analytic solution provides only a coarse approximation for altitude and flight-path angle. Velocity, however, is closely approximated by both methods. This indicates that the cost function $J = -V_f$ varies little between these approximations. Thus, the energy loss is accurately predicted by the approximate solutions. In Fig. 3 it is clear that rapid variations are present in the latter part of the trajectory, since collocation closely approximates the flight-path angle history over all but the last interval. This is corroborated by the Loh's term history in Fig. 4.

It is difficult to capture this type of rapid variation with a simple piecewise linear approximation. In addition, Fig. 4 suggests that the aeroassisted orbit transfer problem is, in fact, a singular perturbation problem as discussed in Ref. 4. A more uniformly accurate collocation solution was obtained by dividing the last interval in the $N = 7$ case into 13 subintervals, resulting in $N = 19$ elements. The flight-path angle results obtained with this strategy were essentially identical to the exact solution.

The open-loop control histories for the zero-order analytic solution as well as the $N = 7$ collocation solution are presented in Figs. 5 and 6. Again, the analytical zero-order solution is a poor approximation. The δ history is approximated quite well with a moderate number of collocation elements. The σ history obtained from collocation differs only slightly from the exact result, a difference which tends to vanish with increasing N.

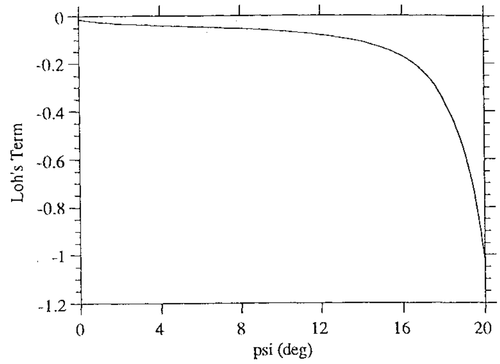


Fig. 4 Optimal Loh's term.

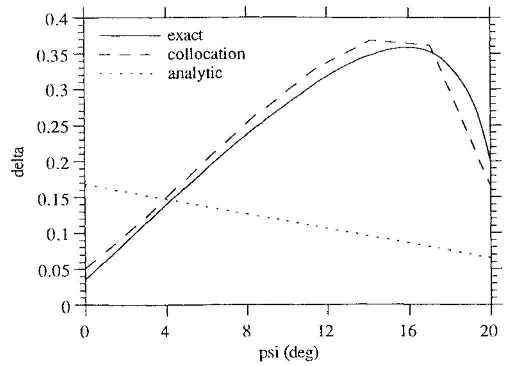


Fig. 5 Open-loop δ profiles.

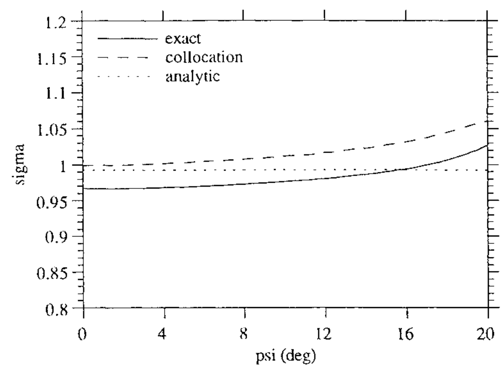


Fig. 6 Open-loop σ profiles.

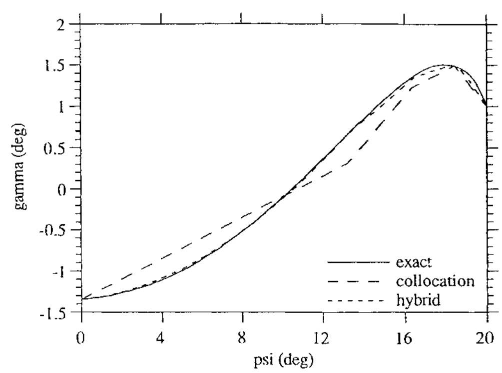


Fig. 7 Open-loop γ comparison plot.

Some open-loop results obtained by pure collocation and by the hybrid method are presented in Figs. 7 and 8. These plots show how, for a given element spacing, the use of nonlinear interpolating functions based on the analytic zero-order solution can result

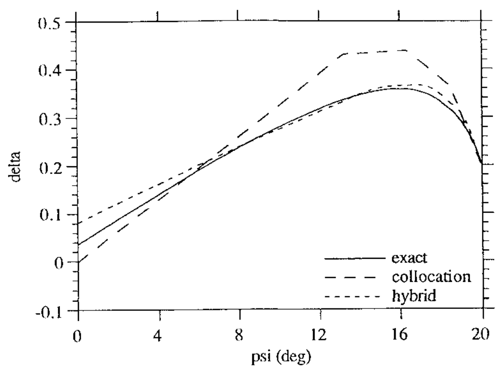


Fig. 8 Open-loop δ comparison plot.

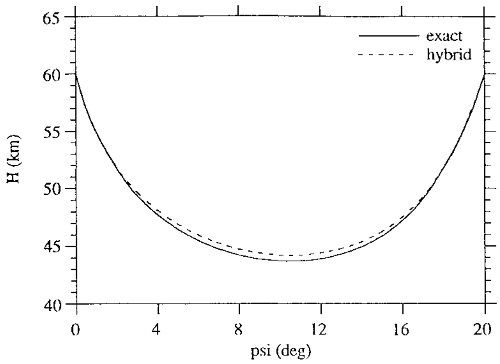


Fig. 9 Optimal and guided H solutions.

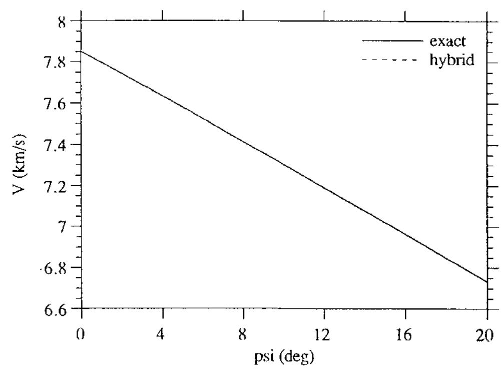


Fig. 10 Optimal and guided V solutions.

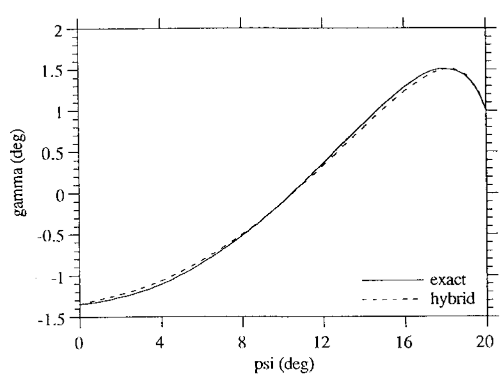


Fig. 11 Optimal and guided γ solutions.

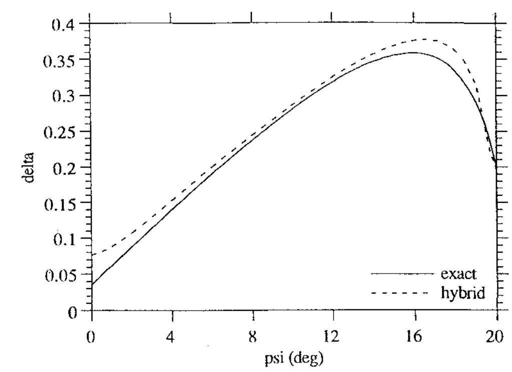


Fig. 12 Optimal and guided δ solutions.

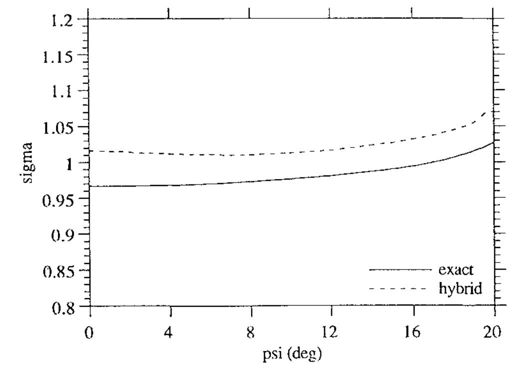


Fig. 13 Optimal and guided σ solutions.

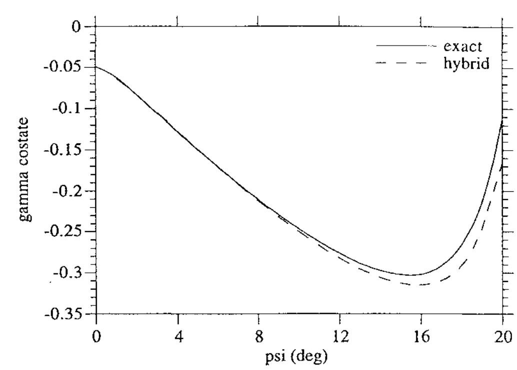


Fig. 14 Optimal and guided λ_γ solutions.

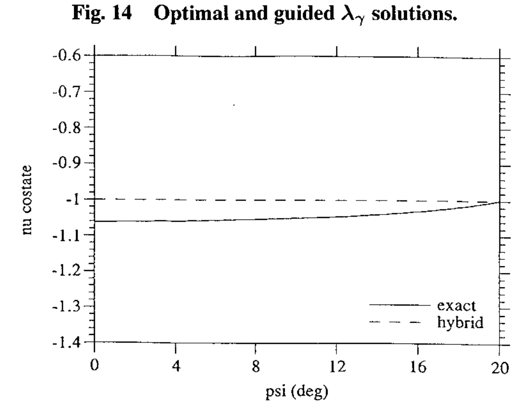


Fig. 15 Optimal and guided λ_ν solutions.

in increased accuracy while reducing computational effort. In these figures, the maneuver is partitioned into four unequal elements with nodes clustered in the final seven degrees of plane change. Additional nodes are necessary if the accuracy of the pure collocation solution is to approach that of the hybrid solution. The nonlinear interpolating functions obtained from the analytical zero-order solution are highly effective early in the maneuver, whereas rapidly varying dynamics later in the trajectory necessitate smaller elements in this region.

Figures 9–15 depict closed-loop results obtained by the hybrid method with four uniformly spaced elements. This corresponds to solving a system of 16 simultaneous nonlinear equations at each control update, a significant improvement over collocation with linear interpolating functions which would require the solution of 33 simultaneous equations. Computations were carried out on a Sun SPARCserver 630MP, and the simulation was completed in 3.5 CPU s. The closed-loop state histories shown in Figs. 9–11 conform closely to the optimal results, as does the δ control history of Fig. 12.

In Fig. 13, the control σ exhibits a deviation of approximately 4% from the true optimum, consistent with the results depicted in Fig. 6. This may be due in part to the fact that λ_ν is approximated as a constant throughout the trajectory. Figure 14 shows the history of λ_γ, an important costate.

Discussion

The MRRV example illustrates the ability of the hybrid approach to achieve computational efficiency by incorporating analytically tractable portions of the dynamics in the choice of interpolating functions. In the example it was shown that the hybrid approach can achieve greater accuracy than piecewise linear collocation while involving fewer unknown parameters. Although collocation with more general nonlinear interpolating functions such as cubic splines might achieve a similar improvement in accuracy, the numerical complexity of the problem would be increased. The introduction of additional unknown parameters would necessitate the real-time solution of a larger system of simultaneous nonlinear equations. In the hybrid approach, more complicated interpolating functions are employed although the number of unknowns is actually decreased. This is made possible by using the analytically tractable portion of the solution to determine the interpolating functions.

In developing the interpolating functions, several options are available. In this paper, λ_ν was treated as constant and, therefore, equal to its natural terminal boundary value of -1.0. The results suggest that this is the main source of error in the hybrid solution. Figure 15 shows that an affine function of z would more closely approximate λ_ν. Thus, the accuracy could be improved by simply adding a new constant q_{ν_j} to the λ_ν dynamics in Eq. (38). This would introduce $N + 1$ new unknowns [$\lambda_\nu(z_0)$ and q_{ν_j}], with the natural boundary condition and N collocation constraints on λ_ν providing the corresponding $N + 1$ equations. However, since the controls σ and δ depend inversely on λ_ν, the new approximation would lead to complicated logarithmic interpolating functions rather than the simple polynomials presented in the Appendix. One attractive feature of the hybrid approach is that specific aspects of the dynamics may either be included analytically or corrected numerically. The most appropriate choice depends on many factors including speed, accuracy, and reliability of the ultimate implementation.

An alternative to improving the accuracy of the hybrid approximation is the calculation of a first-order correction, which was carried out in Ref. 5. Since the aeroassisted plane change problem should actually be formulated as a problem in matched asymptotic expansions, it is of interest to extend this approach in that direction. This suggests that to zero order, two properly matched hybrid solutions would significantly reduce the number of nodes required to accurately represent the solution in the exit phase of the maneuver where Loh's term undergoes a rapid variation.

Conclusions

Several approximate methods are available for determining near-optimal control laws for aeroassisted plane change maneuvers with minimum energy loss. An analytical solution which neglects Loh's term is not sufficiently accurate without higher order corrections. A purely numerical collocation solution requires an excessively large number of finite elements for sufficient accuracy, making this approach computationally undesirable. A hybrid methodology equivalent to collocation with nonlinear interpolation functions based on the zero-order analytic solution may be employed for greater computational efficiency. Since the hybrid approach is formulated piecewise as a regular perturbation problem, its accuracy may be further improved by including a first-order correction.

Appendix: Interpolating Functions

The hybrid formulation of Eq. (38) gives rise to the following nonlinear interpolating functions:

$$\lambda_\nu(z) = -1$$

$$\lambda_\psi(z) = \lambda_{\psi_{j-1}}$$

$$\lambda_w(z) = q_{w_j}\Delta z + \lambda_{w_{j-1}}$$

$$\lambda_\gamma(z) = q_{w_j}/2(\Delta z)^2 + (\lambda_{w_{j-1}} + q_{\gamma_j})\Delta z + \lambda_{\gamma_{j-1}}$$

$$\psi(z) = -(E^*/2)\lambda_{\psi_{j-1}}\Delta z + \psi_{j-1}$$

$$\gamma(z) = -\frac{E^*}{2}\left[\frac{q_{w_j}}{6}(\Delta z)^3 + \frac{\left(\lambda_{w_{j-1}} + q_{\gamma_j}\right)}{2}(\Delta z)^2\right] - \frac{E^*}{2}\lambda_{\gamma_{j-1}}\Delta z + p_{\gamma_j}\Delta z + \gamma_{j-1}$$

$$w(z) = \frac{E^*}{2}\left[\frac{q_{w_j}}{24}(\Delta z)^4 + \frac{\left(\lambda_{w_{j-1}} + q_{\gamma_j}\right)}{6}(\Delta z)^3 + \frac{\lambda_{\gamma_{j-1}}}{2}(\Delta z)^2\right] - \frac{p_{\gamma_j}}{2}(\Delta z)^2 - \gamma_{j-1}\Delta z + w_{j-1}$$

$$\nu(z) = \nu_{j-1} - \left[\frac{1}{E^*} + \frac{E^*}{4}\lambda^2_{\psi_{j-1}}\right]\Delta z - \frac{E^*}{4}\left\{\frac{q^2_{w_j}}{20}(\Delta z)^5 + \frac{q_{w_j}\left(\lambda_{w_{j-1}} + q_{\gamma_j}\right)}{4}(\Delta z)^4 + \frac{\left[q_{w_j}\lambda_{\gamma_{j-1}} + \left(\lambda_{w_{j-1}} + q_{\gamma_j}\right)^2\right]}{3}(\Delta z)^3\right\} - \frac{E^*}{4}\left[\lambda_{\gamma_{j-1}}(\lambda_{w_{j-1}} + q_{\gamma_j})(\Delta z)^2 + \lambda^2_{\gamma_{j-1}}\Delta z\right]$$

where $j = 1, \ldots, N$, $\Delta z = (z - z_{j-1})$, and $z \in [z_{j-1}, z_j]$.

References

[1]Speyer, J. L., and Crues, E. Z., "Approximate Atmospheric Guidance Law for Aeroassisted Plane Change Maneuvers," *Journal of Guidance, Control, and Dynamics*, Vol. 13, No. 5, 1990, pp. 792–802.

[2]Hull, D. G., Giltner, J. M., Speyer, J. L., and Mapar, J., "Minimum Energy-Loss Guidance for Aeroassisted Orbital Plane Change," *Journal of Guidance, Control, and Dynamics*, Vol. 8, No. 4, 1985, pp. 487–493.

[3]Hull, D. G., and Speyer, J. L., "Optimal Re-entry and Plane Change Trajectories," *Journal of the Astronautical Sciences*, Vol. XXX, No. 2, April–June 1982, pp. 117–130.

[4]Calise, A. J., and Melamed, N., "Optimal Guidance of Aero-assisted Transfer Vehicles Based on Matched Asymptotic Expansions," *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, Vol. 2, AIAA, Washington, DC, 1991, pp. 1048–1058.

[5]Leung, S. K., and Calise, A. J., "A Hybrid Approach to Near-optimal Launch Vehicle Guidance," *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, Vol. 1, AIAA, Washington, DC, 1992, pp. 7–21.

[6]Mease, K. D., Lee, J. Y., and Vinh, N. X., "Orbital Change During Hypersonic Aerocruise," *Journal of the Astronautical Sciences*, Vol. 36, Nos. 1/2, 1988, pp. 103–137.

[7]Press, W. H., Flannery, B. P., Teukolsky, S. A., and Vetterling, W. T., *Numerical Recipes (FORTRAN Version)*, Cambridge Univ. Press, New York, 1990, pp. 240–273.

[8]Oberle, H. J., and Grimm, W., BNDSCO, A Program for Numerical Solution of Optimal Control Problems, English translation of DFVLR-Mitt. 85-05, 1985.